

Reconsidering the galactic coordinate system

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Abstract. Initially defined by the IAU in 1958, the galactic coordinate system was thereafter in 1984 transformed from the B1950.0 FK4-based system to the J2000.0 FK5-based system. In 1994, the IAU recommended that the dynamical reference system FK5 be replaced by the ICRS, which is a kinematical non-rotating system defined by a set of remote radio sources. However the definition of the galactic coordinate system was not updated. We consider that the present galactic coordinates may be problematic due to the unrigorous transformation method from the FK4 to the FK5, and due to the non-inertiality of the FK5 system with respect to the ICRS. This has led to some confusions in applications of the galactic coordinates. We tried to find the transformation matrix in the framework of the ICRS after carefully investigating the definition of the galactic coordinate system and transformation procedures, however we could not find a satisfactory galactic coordinate system that is connected steadily to the ICRS. To avoid unnecessary misunderstandings, we suggest to re-consider the definition of the galactic coordinate system which should be directly connected with the ICRS for high precise observation at micro-arcsecond level.

Key words. Astrometry — Galaxy: general — Reference system

1. Introduction

For studies of the galactic structure, kinematics and dynamics, it is important to establish the galactic coordinate system having its equatorial plane in the galactic plane, and the pole coincide with the galactic pole. The definition of the system of galactic coordinates, which is based on the FK4 system at the epoch of B1950.0, was announced by the International Astronomical Union Sub-Commission 33b in the late 1950s (Blaauw et al. 1960). It is a heliocentric rectangular system with three principal axes in the direction of galactic center, galactic rotation, and north galactic pole. Following the recommendation of the Sixteenth General Assembly of the IAU at Grenoble in 1976, the previous fundamental reference system FK4 have been replaced with FK5 at the epoch of J2000.0. In addition, the IAU adopted the new system of astronomical constants and time unit in order to construct an inertial framework from observed materials. The transformation procedures from the FK4 based reference system to the FK5 reference system have been developed by Standish (1982) and Aoki et al. (1983). The transformation matrix from the equatorial coordinate system to galactic coordinate system at the epoch of J2000.0 has been derived by Murray (1989) under the assumption that the FK4 and FK5 systems have only orientation offsets due to the precession motion from B1950.0 to J2000.0 as well as the equinox bias.

The fundamental celestial reference system for astronomical application is now the International Celestial Reference System (ICRS) as specified in the IAU Res. B2 of 1997. To establish the ICRS as a practical system, the IAU provided a set of distant benchmark radio sources, by whose coordinates the direction of the ICRS axes are defined (Ma et al. 1998). The ICRS is realized optically, but at lower accuracy, by the star positions and proper motions of the *Hipparcos Catalogue* (ESA 1997). However, the galactic coordinate system is still based on the observations in the FK4 system at the epoch of B1950.0 or more recently, transformed to the FK5 system in a loose sense. The FK5 system is non-inertial based on the comparison between the FK5 and *Hipparcos* proper motion systems. In other words, the galactic coordinate system, whether its principal axes are in the FK4 system at B1950.0 or in the FK5 system at J2000.0, is rotating with respect to the ICRS. This can lead to some confusions or misunderstandings. Considering a sub-milli-arcsecond or even micro-arcsecond astrometric accuracy, the galactic coordinate system should be directly connected

to the ICRS for uniform practical applications and exact definition of coordinate system. We suggest that a new definition of the galactic coordinate system be considered.

In §2 and §3 of the present work we review the definition of the galactic coordinate system as well as the confusion in astronomical applications. In §4 a suggestion of the new definition is presented and some conclusions are summarized in §5.

2. History of the development of the galactic coordinate system

2.1. IAU1958 galactic coordinate system

From the time of William Herschel in the 1780s until 1932 various galactic coordinate systems which differ from one another by one or two degrees were used. Ohlsson (1932) summarized extensive observations at Harvard Observatory and adopted a position of the north galactic pole at right ascension $\alpha_{B1900.0}^p = 12^h40^m$, declination $\delta_{B1900.0}^p = +28^\circ$. With the development of the radio astronomy in the 1950s, it became evident that the old galactic coordinate system did not serve its purpose any longer and was seriously in error for many observations especially in the radio spectrum. In 1958 the IAU Sub-Commission authorized a new definition of the galactic coordinate system, according to which the equatorial plane was defined by the flat HI principal plane and the zero point of the system by the direction of the galactic center. Radio continuum observations gave strong support to a pole based on the HI observations and optical data were used for a checking purpose. The Sun is effectively defined to be in the plane even though it is thought to be some 10-20 pc above the HI principal plane (Zhu 2009). The revision and reconstruction of the galactic coordinate system was presented in a series of five papers (Blaauw et al. 1960; Gum et al. 1960; Gum & Pawsey 1960; Blaauw 1960; Oort & Rougoor 1960). The system was defined by three principal values:

$$\begin{aligned}\alpha_{B1950.0}^p &= 12^h49^m \\ \delta_{B1950.0}^p &= +27^\circ.4 \\ \theta_{B1950.0} &= 123^\circ,\end{aligned}\tag{1}$$

where $\alpha_{B1950.0}^p$, $\delta_{B1950.0}^p$ are the right ascension and declination of the north galactic pole and $\theta_{B1950.0}$ is the position angle of the galactic center from the equatorial pole. The three principal values defining the galactic coordinate system are based on the FK3 system because at that time in 1958 the FK4 catalogue was not published yet. Nevertheless, the equinox and equator were not changed in the FK4 with respect to the FK3, so it should be justified to assume that the defining values are based on the equator and equinox of the FK4 system at the epoch of B1950.0. These numbers in Equation (1) should be used as a strict definition with infinite accuracy, or in other words, valid to an infinite number of decimals. Users are supposed to quote the definition correctly rather than copy computer algorithms in order to avoid inconsistency in the forward-backward conversion between equatorial (α , δ) and galactic (ℓ , b) coordinate systems (Lane 1979).

Note that the establishment of the galactic coordinate system does not require the same degree of precision as the equatorial reference system which are realized by the fundamental catalogues (FK4, FK5), because it is constructed based on the concentration of HI gas in statistical sense. Although the requirements for the establishment of the galactic coordinate system are less drastic, it must be fixed in the fundamental reference system of the time, say, aligned with FK4, FK5, or ICRS.

2.2. Principal galactic axes in the FK5 system at J2000.0

The introduction of the new fundamental stellar catalogue FK5 and the IAU(1976) system of astronomical constants were brought into effect from 1 January 1984. The transformation from the FK4 to the FK5 includes the following changes:

- the E-term of aberration was removed from the mean places and proper motions;
- the Newcomb’s precession constants were replaced by the IAU 1976 astronomical constants (Lieske et al. 1977);
- the bias between FK4 equinox and the dynamical equinox (equinox correction) was eliminated;
- the time unit was changed from tropical to Julian centuries;
- the standard coordinate system was precessed from the equinox and equator at B1950.0 to that of J2000;
- the positions of the stars were presented at the epoch of J2000.0 by adding their proper motions.
- the systematic differences ($FK5 - FK4$) were corrected, if known.

The so-called E-term of aberration depends explicitly on the eccentricity of the Earth orbit around the Sun and represents the components of the displacement due to the departure of the elliptic orbital motion from a circle. During the process of transforming the galactic coordinates from the FK4 system (B1950.0) to the FK5 system (J2000.0),

the impure rotation related to the E-term of aberration have consequently lead to an unacceptable non-orthogonal system of axes if those axes are considered to be defined by specific objects in the three principal directions (Murray 1989). Accordingly Murray proposed that the axes of IAU1958 galactic coordinate system be considered as absolute directions, unaffected by the E-term of aberration.

In order to understand the transformation, we consider the column unit vector (3×1) of a certain absolute direction or a fictitious distant extragalactic source at the epoch of B1950.0, \mathbf{A}_B and \mathbf{G}_B , and at the epoch of J2000.0, \mathbf{A}_J and \mathbf{G}_J . For brevity, the subscript B stands for the epoch of B1950.0 and J stands for J2000.0. The characters \mathbf{A} and \mathbf{G} denote the unit vectors in the equatorial and galactic systems thus they can be expressed as :

$$\begin{aligned}\mathbf{A}_B &= (\cos \alpha_B \cos \delta_B \quad \sin \alpha_B \cos \delta_B \quad \sin \delta_B)^T \\ \mathbf{A}_J &= (\cos \alpha_J \cos \delta_J \quad \sin \alpha_J \cos \delta_J \quad \sin \delta_J)^T \\ \mathbf{G}_B &= (\cos \ell_B \cos b_B \quad \sin \ell_B \cos b_B \quad \sin b_B)^T \\ \mathbf{G}_J &= (\cos \ell_J \cos b_J \quad \sin \ell_J \cos b_J \quad \sin b_J)^T.\end{aligned}$$

Let \mathcal{N}_B and \mathcal{N}_J be the 3×3 matrixes that convert the position vector from the equatorial system to the galactic system, i.e.

$$\mathbf{G}_B = \mathcal{N}_B \mathbf{A}_B \quad (2)$$

$$\mathbf{G}_J = \mathcal{N}_J \mathbf{A}_J. \quad (3)$$

Suppose that $\mathcal{X}(t)$ is the transformation matrix of position vector from J2000.0 system to that of B1950.0 system at some epoch of t , $\mathcal{X}(t)$ can be computed from the following formula:

$$\mathcal{X}(t) = \mathcal{P}_{NEWC}(t, B1950.0) \mathcal{R}_3(E(t)) \mathcal{P}_{IAU76}(J2000.0, t) \quad (4)$$

where t is reckoned from B1950.0 in tropical centuries. $\mathcal{P}_{IAU76}(t_1, t_2) = \mathcal{R}_3(-z_A) \mathcal{R}_2(\theta_A) \mathcal{R}_3(-\zeta_A)$ is the precession matrix from the epoch of t_1 to t_2 based on IAU (1976) constant system and $\mathcal{P}_{NEWC}(t_1, t_2)$ the precession matrix based on Newcomb's precession formulae. The precession matrices were constructed for the transformation of coordinates from one epoch (t_1) to another (t_2). For the equinox correction, Fricke (1982) pointed out that the equinox correction $E(t)$ at any epoch can be expressed as

$$E(t) = E_{1950} + \dot{E}t. \quad (5)$$

We assume here that $E = 0''.525$ at the epoch of J1950.0 and $\dot{E} = 1''.275/\text{Julian century}$ as Aoki et al. (1983) did for the consistency of the new formula for the sidereal time.

According to Sôma & Aoki (1990)'s method, the E-term in aberration and the systematic and individual corrections are ignored, thus the equatorial position is only affected by the precession motion from B1950.0 to J2000.0 and the equinox correction:

$$\mathbf{A}_J = \mathcal{X}_0^{-1} \mathbf{A}_B \quad (6)$$

where the matrix \mathcal{X}_0 is the constant term in the expanded polynomial of $\mathcal{X}(t)$:

$$\mathcal{X}(t) = \mathcal{X}_0 + \mathcal{X}_1 t + \mathcal{X}_2 t^2 + \dots$$

We can see from Equation (6) that in current problem the transformation is unrelated to the epoch t .

Because the galactic coordinate system must be regarded absolute in the celestial sphere, the galactic coordinates (ℓ, b) of a given direction (or a distant radio source) will not change although its equatorial coordinates are referred to the FK4 system and then transformed to the FK5 system. The transformation only takes the orientation difference between the FK4 and the FK5 into account, as if they were both inertial. Hence we have

$$\mathbf{G}_J = \mathbf{G}_B. \quad (7)$$

Finally we obtain from Equation (3), (6) and (7) the transformation matrix based on the FK5 system at the epoch of J2000.0:

$$\mathcal{N}_J = \mathcal{N}_B \mathcal{X}_0. \quad (8)$$

The numerical matrices, printed to 0.1 milli-arcsecond accuracy of the precession formulae in the IAU (1976) system, are

$$\mathcal{N}_B = \begin{pmatrix} -0.066988739410 & -0.872755765850 & -0.483538914637 \\ +0.492728466081 & -0.450346958020 & +0.744584633279 \\ -0.867600811149 & -0.188374601732 & +0.460199784785 \end{pmatrix};$$

$$\mathcal{X}_0 = \begin{pmatrix} +0.999925679496 & +0.011181483239 & +0.004859003772 \\ -0.011181483221 & +0.999937484893 & -0.000027170294 \\ -0.004859003815 & -0.000027162595 & +0.999988194602 \end{pmatrix},$$

and

$$\mathcal{N}_J = \begin{pmatrix} -0.054875539390 & -0.873437104725 & -0.483834991775 \\ +0.494109453633 & -0.444829594298 & +0.746982248696 \\ -0.867666135681 & -0.198076389622 & +0.455983794523 \end{pmatrix}. \quad (9)$$

Such a procedure guarantees that the galactic coordinate system is orthogonal and non-rotating with respect to the FK5 system at the epoch of J2000.0.

Inserting ($b = 90^\circ$) and ($\ell = 0, b = 0$), respectively, in Equation (3), we can obtain the direction of the galactic pole and the position angle in the J2000.0 equatorial system (also at 0.1 milli-arcsecond level):

$$\begin{aligned} \alpha_J^p &= 12^h 51^m 26^s.27549 \\ \delta_J^p &= +27^\circ 07' 41''.7043 \\ \theta_J &= 122^\circ.93191857, \end{aligned} \quad (10)$$

and the equatorial coordinates of the galactic center:

$$\begin{aligned} \alpha_J^0 &= 17^h 45^m 37^s.19910 \\ \delta_J^0 &= -28^\circ 56' 10''.2207. \end{aligned} \quad (11)$$

Murray (1989) and Miyamoto & Sôma (1993) have also obtained coincidental results using a slightly different method with only a few tenth of milli-arcsecond discrepancy. It is worth noting that if we set $t = 0$ (i.e. transformation at the epoch of B1950.0) in Equation (4) and its expanded form, the transformation matrix is given by :

$$\mathcal{X}_0 = \mathcal{R}_3(E(B1950.0))\mathcal{P}_{IAU76}(J2000.0, B1950.0), \quad (12)$$

which was adopted by Murray (1989).

The transformation between equatorial and galactic coordinate system in the *Hipparcos* system (ESA 1997) adopted Murray's method during the conversion from the equatorial to galactic coordinates in which the difference between the FK5 and the *Hipparcos* is ignored. It is worth noting that the *Hipparcos* transformation matrix is numerically different from ours at the milli-arcsecond level because it is calculated by adopting α_J^p, δ_J^p and θ_J as definitional values. There is possible loss of precision during the computing procedure, as adopted by *Hipparcos*, from the truncated value of $\alpha_J^p, \delta_J^p, \theta_J$ to the matrix \mathcal{N} :

$$\mathcal{N}_{Hip} = \begin{pmatrix} -0.0548755604 & -0.8734370902 & -0.4838350155 \\ +0.4941094279 & -0.4448296300 & +0.7469822445 \\ -0.8676661490 & -0.1980763734 & +0.4559837762 \end{pmatrix}.$$

3. Confusions and misunderstandings in applications of the galactic coordinate system

As mentioned in last sections, the present galactic coordinates may be problematic due to the inaccurate transformation from one fundamental reference system to another. Consequently, some confusion in applications of galactic coordinates and misunderstandings in the concept of the reference system can be found.

3.1. Conversion of galactic coordinates from the FK4 to the FK5 system

According to the IAU resolutions adopted in 1976 (Trans. IAU 16B), in 1979 (Trans. IAU 17B), and in 1982 (Trans. IAU 18B), new methods and astronomical constants system are used for the computation of the positions and proper motions of stars. Conversion processes of mean positions and proper motions of stars from the FK4 system (B1950.0) to the FK5 system (J2000.0) had been proposed by Standish (1982), Aoki et al. (1983) and subsequently reviewed by Smith et al. (1989), Yallop et al. (1989), Murray (1989) and Sôma & Aoki (1990). The methods for calculating the apparent places in the new IAU system were also developed by Lederle & Schwan (1984).

The rigorous method of converting the position and proper motion of a celestial object at B1950.0 to those at J2000.0 contains two major steps: (i) remove the E-terms of aberration and their variations' effect; (ii) transfer the

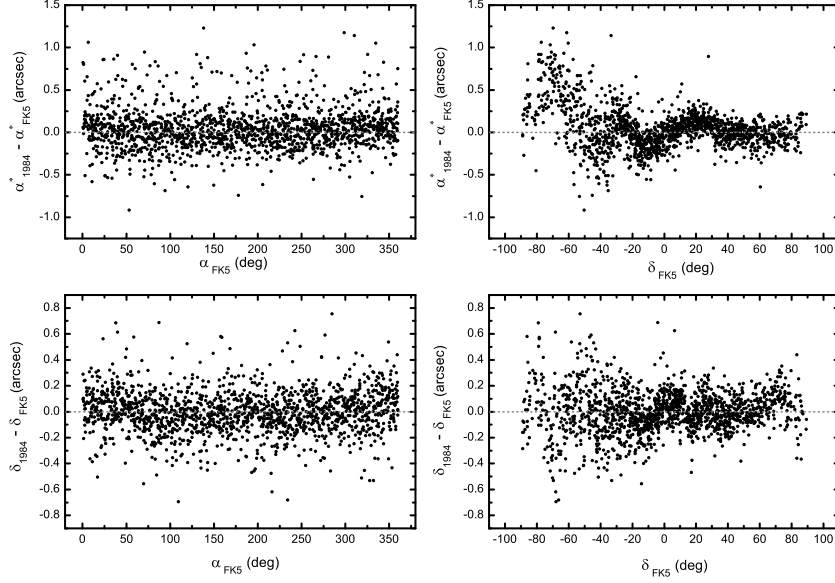


Fig. 1. Position differences between the FK4-based and FK5 bright stars. The subscript 1984.0 indicates the transformation is operated at the epoch of 1984.0 by the matrix $\mathcal{M}_{1984.0}$.

position and proper motion vector using a 6×6 matrix to the new IAU system. Adopting the notations given by Standish (1982), the 6-space transformation matrix operated on 1984 January 1.0 (Aoki et al. 1983) is described as

$$\begin{aligned} \mathcal{M}_{1984.0} = & \begin{pmatrix} I & G_2 I \\ O & I \end{pmatrix} \mathcal{P}_{IAU76}^{-1}(J2000.0, 1984.0, 1) \mathcal{Q}_3(E(1984.0), \dot{E}) \\ & \times \begin{pmatrix} I & O \\ O & FI \end{pmatrix} \mathcal{P}_{NEWC}(B1950.0, 1984.0, 1) \begin{pmatrix} I & G_1 I \\ O & I \end{pmatrix}, \end{aligned} \quad (13)$$

where G_1 and G_2 are the time intervals $1984.0 - B1950.0$ and $J2000.0 - 1984.0$, respectively. For details of the notions in Equation (13) and the constructing procedure of $\mathcal{M}_{1984.0}$, readers are referred to Standish (1982) and Aoki et al. (1983). The numerical form of our matrix $\mathcal{M}_{1984.0}$ shows consistency with other authors' except the difference in the last digit. Based on the transformation matrix we convert the positions and proper motions of 1535 FK4 bright stars from B1950.0 to J2000.0. Figure 1 displays the position difference of corresponding stars in the FK4 and FK5 catalogue at the epoch of J2000.0. The top panels show the position difference in the sense of $FK4(J2000.0) - FK5$ for right ascensions and the bottom panels for declinations. The position differences in the southern hemisphere are more pronounced because of the lower quality observations of the stars. Besides, the plots show residual systematic difference versus declinations, which imply the imperfection of the transformation procedure from the FK4 to the FK5 reference frame. On the other hand, the differences for most stars are less than 0.3 arcsec in right ascension and declination, which shows the correctness of the transformation in the range of the error of the FK4 system.

During the transformation, a correction of the time derivative of the E-term would produce an additional effect on the proper motions in comparison to that from the pure rotations. In fact, if we assume that the position of an extragalactic object referred to the FK4 (or FK5) system is fixed (because its proper motion is zero), then it has non-zero fictitious proper motion with respect to the FK5 (or FK4) system, by the corresponding total amount of difference in derivative of E-term. Reid & Brunthaler (2004) in their study gave out the coordinates of the north galactic pole in J2000.0 system, $\alpha_J^p = 12^h 51^m 26^s.282$, $\delta_J^p = +27^\circ 07' 42''.01$, and of the galactic center, $\alpha_J^0 = 17^h 45^m 37^s.224$, $\delta_J^0 = -28^\circ 56' 10''.23$, as well as the position angle of the direction of the galactic center, $\theta_J = 122^\circ.932$ (see their Appendix). We tried to transform the IAU 1958 version of the definition in Equation (1), using the rigorous method (two steps described above), from B1950.0 to that of J2000.0 and found identical values as the results derived by Reid & Brunthaler (2004). As a byproduct, the proper motions of both points are $\mu_\alpha^p = 0.289881$, $\mu_\delta^p = 0.423357$, $\mu_\alpha^0 = -0.003708$ and $\mu_\delta^0 = 0.019808$ in as ct^{-1} which can be attributed to the derivative of the E-term in aberration. Such a transformation procedure would be problematic for the purpose of calculating the orientation of the new galactic coordinate system. Firstly, the x - and z - axes, pointing to the galactic center and north galactic pole,

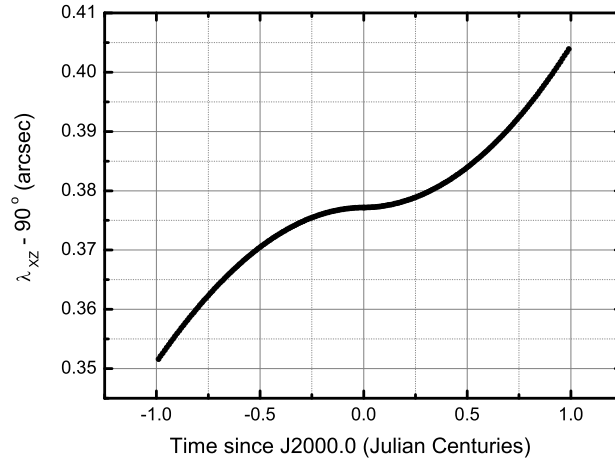


Fig. 2. The relationship between $\lambda_{xz} - 90^\circ$ and time interval since J2000.0. The symbol λ_{xz} is the angular separation between the x - and z -axes, which is computed backward and forward from J2000.0 using constant proper motion $\mu_\alpha^p = 0.289881$, $\mu_\delta^p = 0.423357$, $\mu_\alpha^0 = -0.003708$ and $\mu_\delta^0 = 0.019808$ in as cty^{-1} .

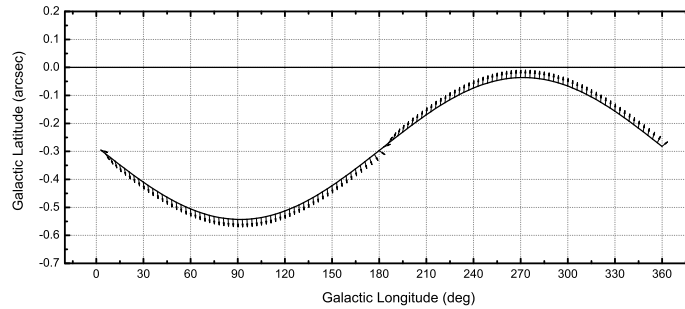


Fig. 3. The distortion of the galactic plane (the solid curve) in the FK5 system at J2000.0. The vectors denote the proper motion of certain points although they should be fixed in the FK4 system at B1950.0. The solid line is the nominal galactic plane with $\ell = 0$.

are not perpendicular to each other with a mismatch of 0.377 arcsec at the epoch of J2000.0. The discrepancy will accumulate as the time interval gets longer (see Figure 2). Secondly, the points like the intersections of the coordinate axes and celestial sphere, and circles like the galactic equator, which must be fixed in the FK5 system at J2000.0 are going to move at the rate of their “fictitious proper motions”. Figure 3 describes the distortion of the galactic plane in the FK5 reference system if it is transformed from B1950.0, when it was a rigorous great circle on the celestial sphere. The vectors denote the “proper motions” coming from the non-pure-rotation transformation. The reduced galactic plane is a time-dependent curve. This incoherence is attributed to the adoption of definitional values, like in Equation (1), as the coordinates of specific objects.

On the other hand, Equation (4) is not a rigorous transfer equation from the FK4 to the FK5 system, which would cause confusion in observations in the galactic coordinate system. For instance, supposing an object was observed to coincide with the galactic center at the epoch of B1950.0 in the FK4 system ($\ell_B = 0$, $b_B = 0$), its galactic coordinates cannot correspond with the galactic center at J2000.0 ($\ell_J \neq 0$, $b_J \neq 0$), if the equatorial position was transformed using the rigorous method from B1950.0 to J2000.0 and then using \mathcal{N}_J from equatorial to galactic coordinate system at the epoch of J2000.0. In other words, the direction ($\ell_J = 0$, $b_J = 0$) would not correspond to the original object in the galactic center. Any other stars on the celestial sphere will suffer this effect (tenth of arcsecond level) if related to the transformation of the galactic coordinates from the FK4 to FK5 system. Therefore the transformed galactic coordinate system (Murray 1989) does not coincide with the physical features as in the 1958 definitions. One may have doubt where the actual galactic center and galactic plane are. For other applications of the conversion, Aoki et al. (1986) had discussed the optical observation and the difficulties in observing the radio sources. According to their study, specified reference framework and constants system are essential to avoid unnecessary confusion.

3.2. Conversion of galactic coordinates from the FK5 system to the ICRS

The establishment of the ICRS as the fundamental celestial reference system was described by Arias et al. (1995), and Feissel & Mignard (1998). It is a coordinate system whose origin is at the barycenter of the Solar system and whose axis directions are effectively defined by the adopted 212 extragalactic radio sources observed by VLBI technique (Ma et al. 1998). Although, more recently, the ICRF-2 (Boboltz et al. 2010) which includes 3414 extragalactic radio sources was put into effect on January 1, 2010, the definition of the ICRS was not changed. The orientation of the ICRS was designed to be consistent with the dynamical equator and equinox (realized by the FK5 catalogue) at the epoch of J2000.0.

Systematically, a *frame bias matrix* \mathcal{B} is needed to convert ICRS data to the dynamical equator and equinox of J2000.0. It is a fixed set of very small rotations and can be used as follows:

$$\mathbf{A}_J = \mathcal{B} \mathbf{A}_{ICRS} \quad (14)$$

where the column vector \mathbf{A} has the similar form as before except for the subscripts which denote the benchmark reference system or epoch. The matrix \mathcal{B} is, to the first order (IERS Conventions 2003),

$$\mathcal{B} = \begin{pmatrix} 1 & d\alpha & -\xi_0 \\ -d\alpha & 1 & -\eta_0 \\ \xi_0 & \eta_0 & 1 \end{pmatrix} \quad (15)$$

where the angles $\eta_0 = -6.819 \pm 0.0015$ mas and $\xi_0 = -16.6171 \pm 0.0080$ mas are the ICRS pole offsets in x, y directions, and $d\alpha = 14.6 \pm 0.5$ mas is the offset in the equinox between the ICRS and J2000.0 dynamical system (Chapront et al. 2002). See Hilton & Hohenkerk (2004) for detailed description. To the second order, the more precise format is

$$\mathcal{B} = \begin{pmatrix} 1 - (d\alpha_0^2 + \xi_0^2)/2 & d\alpha_0 & -\xi_0 \\ -d\alpha_0 - \eta_0 \xi_0 & 1 - (d\alpha_0^2 + \eta_0^2)/2 & -\eta_0 \\ \xi_0 - \eta_0 d\alpha_0 & \eta_0 + \xi_0 d\alpha_0 & 1 - (\eta_0^2 + \xi_0^2)/2 \end{pmatrix}. \quad (16)$$

The above matrix can also be used to transform vectors from ICRS to the FK5 system at the epoch of J2000.0 by simply substituting $d\alpha_0 = -22.9$, $\eta_0 = -19.9$ and $\xi_0 = 9.1$ mas (Mignard & Froeschlé 1997).

For the galactic coordinate system, we can use Murray's method for the purpose of constructing the equatorial-galactic transformation matrix in the ICRS. Neglecting the relative rotation between the FK5 system and the ICRS and assuming the axes of the galactic coordinate system are absolute, we get the relation between \mathcal{N}_J and \mathcal{N}_{ICRS} :

$$\mathcal{N}_{ICRS} = \mathcal{N}_J \mathcal{B}. \quad (17)$$

The numerical form of the matrix \mathcal{N}_{ICRS} is:

$$\mathcal{N}_{ICRS} = \begin{pmatrix} -0.054875657707 & -0.873437051953 & -0.483835073621 \\ +0.494109437203 & -0.444829721222 & +0.746982183981 \\ -0.867666137554 & -0.198076337284 & +0.455983813693 \end{pmatrix} \quad (18)$$

by adopting \mathcal{B} as in Equation (16). The direction of the galactic pole and the position angle in the ICRS should be

$$\begin{aligned} \alpha_{ICRS}^p &= 12^h 51^m 26^s.27469 \\ \delta_{ICRS}^p &= +27^\circ 07' 41''.7087 \\ \theta_{ICRS} &= 122^\circ.93192526. \end{aligned} \quad (19)$$

The difference between Equation (10) and (19) is at the order of several milli-arcseconds. The above discussion only considered the orientation bias between the FK5 and the ICRS. In the sense of inertia, the galactic coordinates have a slow spin as well as regional distortion with respect to the ICRS due to the imperfections in the FK5 system. Such a derived galactic coordinate system is still an expediency because it is not fixed in the ICRS.

In order to reveal the systematic difference between the FK5 and ICRS, which is the optically realized by the *Hipparcos catalogue*, various authors (Zhu & Yang 1999; Mignard & Froeschlé 2000; Zhu 2000; Walter & Hering 2005) have intensively compared the proper motion systems of the two catalogues. To summarize the studies, the global rotation between the FK5 and ICRS, which is derived from the comparison of the proper motion systems of FK5 and *Hipparcos* show obvious inconsistencies (approximately -1.3 mas yr^{-1} by rotational offsets of the *Hipparcos* and FK5 proper motion systems), compared to that predicted by the precession correction, Δp , and correction of the fictitious motion of the equinox, $\Delta e + \Delta \lambda$, from the unprecedented-accurate VLBI and LLR observations (Charlot et al. 1995; Capitaine et al. 2004). The mismatch is probably caused by the error and nonrigid nature of the FK5 proper-motion system and this strongly implies the existence of non-inertiality of the FK5 system. Directly speaking, the FK5 is

rotating with respect to the distant radio sources even if the precession correction is applied to the IAU (1976) precession constants. Furthermore, the region-, magnitude-, and color-dependent systematic errors between the FK5 and *Hipparcos* prevent us from carrying out a perfect alignment of the FK5 system with the ICRS by using simple rotation matrices. Sôma (2000) analyzed Lunar occultations and found much larger errors in the rotation of the *Hipparcos* reference frame than those given by *Hipparcos* team, which suggested the possibility of a non-inertiality of the *Hipparcos* system. Therefore we come to the conclusion that the galactic coordinate system derived from a transformation is neither space-fixed nor time-independent. The system is rotating with respect to the distant extragalactic objects even if it is exactly fixed to the FK5 system.

4. Reconsidering the definition of the galactic coordinate system

We note that the present galactic coordinate system does not meet the defining criterion of the fundamental reference system, namely, non-rotating with respect to the ensemble of distant extragalactic objects. Using the transformation method we can not tie the galactic coordinate system, which was originally defined in 1958, uniquely to the ICRS. There must be residual rotation (slow spin) due to the non-inertial property of the FK5 system which is unacceptable for high-precision observations at the level of one micro-arcsecond. There actually exist several different galactic coordinate system, on the basis of the FK4, the FK5, or the ICRS, having orientation offsets and relative rotation among one another. The least (relative rotation neglected) differences (between Equation (19) and (10)) in the principal values that define the galactic coordinate system in the sense of *ICRS* – *FK5* are roughly:

$$\begin{aligned}\Delta\alpha^p &= -1.2 \text{ mas} \\ \Delta\delta^p &= -4.4 \text{ mas} \\ \Delta\theta &= +24.1 \text{ mas}.\end{aligned}$$

These offsets are much larger than the axes stability of the ICRS ($\simeq 20\mu\text{as}$), so the old definition is not appropriate in current fundamental reference system. In order to avoid the confusion in choosing the galactic coordinate system on one hand and to clarify the concept of the galactic coordinate system on the other, we think that a new definition is needed. Since more and more important astrometric data, such as UCAC3 (Zacharias et al. 2010), PPMX (Röser et al. 2008) and PPMXL (Roeser et al. 2010) are presented in the ICRS framework, the galactic coordinate system should be directly connected with the ICRS for practical observations and studies of the galactic structure, kinematics and dynamics, especially for the galactic center.

To successfully achieve the objective of establishing a galactic coordinate system fixed to the ICRS, which was originally defined in the radio bandpass, the principal plane and the galactic center should likewise be observed using radio techniques. For example, direct observation of the compact radio source Sagittarius A* (Sgr A*) at the dynamical center of the Milky Way (Reid et al. 2009) might give the starting point of the galactic longitude. With measurements using the Very Long Baseline Array (VLBA), the best estimation of the absolute position (J2000.0) of Sgr A* (Reid & Brunthaler 2004) is

$$\begin{aligned}\alpha &= 17^h 45^m 40^s .0400 \\ \delta &= -29^\circ 00' 28'' .138,\end{aligned}\tag{20}$$

and the proper motion of Sgr A* is: $\mu_\ell = -6.379 \pm 0.026 \text{ mas yr}^{-1}$ and $\mu_b = -0.202 \pm 0.019 \text{ mas yr}^{-1}$ that may reflect the galactic rotation and the peculiar motion of the Sun. The uncertainty of the position ($\pm 10 \text{ mas}$) is dominated by the position of J1745-2820. Considering that the proper motion of Sgr A* is nonzero, the observational position should be reduced to the fixed epoch J2000.0. Suppose that Sgr A*, whose position in the ICRS is given by Equation (20), is at the galactic center (i.e. the Sgr A* is regarded as the zero point of the system), we can find the position of the corresponding galactic pole \mathbf{A}_{New}^p from following equations:

$$\begin{aligned}\mathbf{A}_{New}^p \cdot \mathbf{A}_{New}^0 &= 0 \\ \mathbf{A}_{New}^p \cdot (\mathbf{A}_{New}^0 \times \mathbf{A}_{ICRS}^p) &= 0,\end{aligned}\tag{21}$$

where \mathbf{A}_{ICRS}^p is the unit column vector along the galactic pole derived from Equation (19) and \mathbf{A}_{New}^0 the position vector of the new galactic center defined by Equation (20). This new galactic pole is located closest to the old one (\mathbf{A}_{ICRS}^p). The resulting coordinates of the new galactic pole in the ICRS are (printed to one micro-arcsecond accuracy):

$$\begin{aligned}\alpha_{New}^p &= 12^h 51^m 36^s .7151981 \\ \delta_{New}^p &= +27^\circ 06' 11'' .193172.\end{aligned}\tag{22}$$

Accepting the coordinates in Equation (20) as a new definition of the zero point of the galactic coordinate system, and the position described in Equation (22) as the the position of the new galactic pole, we obtain the transformation matrix:

$$\mathcal{N}_{New} = \begin{pmatrix} -0.054657353964 & -0.872844082054 & -0.484928636070 \\ +0.494055821648 & -0.445679169947 & +0.746511167077 \\ -0.867710446378 & -0.198779490637 & +0.455593344276 \end{pmatrix}, \quad (23)$$

as well as the position angle:

$$\theta_{New} = 123^\circ.0075021536. \quad (24)$$

These values are not consistent with those in previous sections because we adopted different coordinates for the galactic center which have several arcminutes disagreement.

As for the formal definition of the galactic reference system in the ICRS, re-observation of the galactic center and the HI principal plane in the radio bandpass is necessary and multi-wavelength observations may be needed for practical application purposes. Because the height of the Sun above the galactic plane z_0 is not zero, the equatorial plane of the galactic coordinate system, which should theoretically pass through the Sun and the galactic center, should be set as close to the HI principal plane as possible. Then the obtained galactic orientation parameters should be regarded as the definition of the new galactic coordinate system. Since the galactic coordinate system is fixed in the ICRS, there is no epoch associated with a galactic coordinate, so it is technically incorrect to say that the galactic coordinate system is a “J2000.0 system”.

As the newly defined galactic coordinate system should become effective, the data in old (FK5) galactic coordinate system could be calculated at the epoch of J2000.0 by multiplying the bias matrix \mathcal{B} where high accuracy is not required. In the optical frequency range, the new galactic coordinate system should be fixed in the *Hipparcos* celestial reference system (HCRS) which is aligned to the ICRS using various methods (Kovalevsky et al. 1997). The estimated uncertainty of the link corresponds to a standard error of 0.6 mas in axes alignment and 0.25 mas yr^{-1} in the rate of rotation.

The galactic coordinate system that we considered is a convenient barycentric system. If we know the accurate distance of the Sun out of the galactic plane, one could consider rotating the system to that which would be observed by an observer exactly in the plane. This would be approximately a 0.1 degree (the angle of the Sun with respect to to the galactic plane viewed from the galactic center) rotation so that the origin of the reference system is certainly in the galactic plane. Kovalevsky (2003) pointed out that a galactocentric rather than a barycentric reference frame should be used because upcoming space astrometric missions Gaia (ESA 2000) and SIM (Danner & Unwin 1999) will drastically improve the astrometric accuracy to micro-arcsecond level. If one wishes to define a reference system centered at the galactic center, one need to know the exact galactic constants like the galactocentric distance of the Sun (R_0), Oort constants (A , B), the vertical distance of the Sun above the galactic plane (z_0) and any other parameters that define the position and the motion of the Sun. However all these values are estimated and still open for research; there is no official resolution about the definite values of galactic constants — different galactic constants would lead to different reference systems. The definition of a galactocentric reference system is at risk and uneconomical even if the galactic constants are authorized because all observations were made in a barycentric system. The astrometric data (positions, proper motions and parallaxes of celestial bodies) must be converted from barycentric to Galactocentric reference system by some more complicated transformation which are related to the galactic constants, rather than ordinary rotation once such coordinate system is defined.

5. Conclusion

We have reviewed the history of the galactic coordinate system, which was primitively defined by the International Astronomical Union in 1958. The efforts of transforming the galactic coordinate system from the FK4 to the FK5, and from the FK5 to the currently used fundamental reference system – ICRS, have led to some confusions and misunderstandings. The galactic coordinate system is rotating with respect to the ICRS and the effect can not be eliminated by simple rotation due to the complexity of the fundamental catalogues (FK4, FK5). As a temporary expedient, we have derived the rotation matrix from the equatorial to the galactic coordinate system in the framework of ICRS using the bias matrix, and this can be used only when high accuracy is not needed. Our proposal for the new definition of the galactic coordinate system makes use of current observation of the supermassive black hole Sgr A* and is, of course, still open for discussion. With the development of radio astronomy to unprecedented accuracy, we suggest that the galactic coordinate system be directly connected to the ICRS by observing the galactic center and plane. Therefore, it is recommended that the definition of the galactic coordinate system be reconsidered by the relevant IAU Commissions.

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